Barbers & Big Ideas: Paradox in Math and Poetry

by Alice Major

Abstract

Paradox intrigues both mathematicians and artists of all kinds. Throughout the recorded history of human thought, paradox has been a signal that we have to look hard for explanations, whether in natural language or the symbols of math and logic. The particular resonance between math and poetry is related to the fact that paradoxical concepts can translate from one form of expression to another surprisingly well. The author examines paradoxes that have intrigued her and presents five of her poems that have been inspired as a result.

1. Russell and Zeno

Paradox has long been a mental sandbox in which mathematicians, logicians and poets like to play around. A paradox is something that is both true and not true, simultaneously logical and illogical. Its contradictions can seem trivial, nonsensical, and yet lead on to powerful insight. And it serves as a hinge between two modes of thought—mathematical and artistic—that are often considered different.

Perhaps this is the case because, at heart, mathematics and art are deeply paradoxical activities. "The paradox of the arts is that they are all made up and yet they allow us to get at truths about who and what we are or might be…" wrote poet Seamus Heaney [10, p 69]. Mathematics shares this quality of being "made up" and yet deeply true as a way of describing the world's relationships.

Paradox is perhaps most familiar in the visual arts—think of Escher's reversing staircases [8], where components that are locally true are globally inconsistent. The human visual system is prone to getting stuck in paradoxical constructions like the Necker cube [19], in which we can't resolve the contradictions but flip back and forth between two equally logical 'truths' deduced from the premises presented by the drawing. Such visual paradoxes pack a punch and intrigue us right away.

The triangle joining math, poetry and paradox is different and perhaps more subtle. However, it is equally close, based on the fact that poetry is a language-based art and shares features with the 'language' aspects of mathematics. A proof is a kind of narrative that goes from premise to conclusion, and (like

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poems), the best proofs have an elegant concision that illuminates. Generally speaking, though, it isn't all that easy to go from math to natural language. A typical proof follows the conventions and symbolic patterns of its mathematical subject—it *can* be expressed in day-to-day speech, but the translation is cumbersome and time-consuming. However, important paradoxes can often be expressed quite easily in the words and narrative of everyday life.

Take, for example, Russell's famous paradox, about a set that is made up of all the sets that cannot be members of themselves. The contradictory nature and consequences of that concept can be laid out in the symbols of math: let $R = \{x \mid x \notin x\}$, then $R \in R \Leftrightarrow R \notin R$. This string of symbols expresses an untenable conclusion: that two mutually exclusive outcomes are materially the same thing. However, for non-mathematicians, Russell's contradiction can be expressed in the story of the Barber's Paradox: Imagine that there's a town with only one barber who has made it a rule that he will only shave those who never shave themselves. So, who shaves the barber? If he shaves himself, he *cannot* shave himself.

The consequences can be expanded as follows:

The Barber's Paradox

The Barber's Paradox is very bad for business.
"I'm only going to shave you if you never shave yourself,"
says the sign on the shop door, below the striped pole
where red and white revolving lines chase themselves silly
into spirals. "But it's Saturday," protests the customer
whose beard bristles insistently through his skin.
"I'd like a little luxury today—don't usually have time
for nice hot towels and that lovely lather, scented
with coconut and lime, on your swirling brush."

But the barber is implacable—he will never shave
those who shave themselves. He is a man of principle.
His own beard foams across his chest, billows
over the shiny leather of the chairs, fills up
the shop, because of course, he cannot shave
any man who ever shaves himself. The customer
grouses, "Well anyway, you'd never find the razor
under all that hair." The doorbell tinkles. The spiral pole revolves.
The barber, somewhat sadly, thinks it's hard to pay the rent
when your theories are set.

— Alice Major [13]

My poem is light-hearted, of course, but Russell's paradox is not merely a logical game. It seriously shook up the foundations of set theory in the early 20th century. It's an example of how an apparently minor question of definitions has significant consequences for the consistency of a self-contained system [2].

I find paradoxes most congenial to poetry, however, not when they point up the failure of a system of logic but because so often they illuminate the 'big' ideas—the nature of truth and falsehood, space and time, and the necessary embrace of contradiction that seems inherent in life. These have been central themes of generation after generation of poets. Take John Donne's metaphysical poems as one example, where his lovers are "stiff twin compasses"—two apparently separate souls that are, in fact, one instrument [7]. Or, more recently, Wallace Stevens' image of "Nothing that is not there and the nothing that is" [16, p 12].

Classic riddles that particularly intrigued me when I was young were those of Zeno. In his first paradox, motion is impossible because to cross any distance means you have to cross half of it first, then half of the remainder and so on. In his third, a variation of the first, he argues that an arrow could never reach its target [3, pp 20 - 22]. I was fascinated because, at ten or twelve and having mastered some basic fractions, I couldn't figure out the flaw in the logic. And yet we clearly lived in a world where I ran across rooms and streets and threw balls at walls.

Of course, grade four fractions are inadequate for handling motion mathematically. It took two thousand years and the development of calculus by Newton and Leibniz (and later, Cantor's transfinite arithmetic) [15, loc. 524] to develop the necessary tools. Today, Zeno's paradox seems like a quaint mathematical take on our moving world, but it still illuminates the nature of continuity and the constant cognitive flip we must make between considering something as an ongoing process or as a completed act. In math terms, the distance from zero to one represents a single completed step in the infinite series of integers to follow. At the same time, within that single step's distance lie mathematical infinities— infinities made up of fractions (and rational and irrational numbers). Mathematically, Zeno's paradox can be resolved by observing that the addition of the infinite series of midpoint distances sums up to the finite total distance, that is: $1/2 + 1/4 + 1/8 + 1/16 + \ldots = 1$. Of course this resolution had to wait till mathematicians discovered the notion of *a limit*, which made infinite summation possible.

The same tension is felt in human lives, as the following sonnet endeavors to illuminate. Life and consciousness are "continuous" experiences, like the number line, and yet an individual life has a discontinuous, integer-like end to consciousness—a limit that is reached, and yet never *quite* reached.

Zeno's paradox

We've solved the paradox.
Motion is possible. The arrow's flight ends
even if its fractions interlock
to infinity—half a distance, yet again
half, and half …. We know this series sums
to a finite thunk and shudder.
                    And we know

the thrumming calculus of life comes
to completion. I am half-way through
my count of years—half-way to knowing
all I will know. Yet something stalls
in the air, an infinitely subtle slowing.
Of whatever I have learned when the arrow falls
    silent, one last sliver will be lost.
    A final distance will remain uncrossed.

           — Alice Major [13]

Like so many others, Zeno's paradoxes are tangled up with ideas about infinity—one of those big ideas that intrigue poets as well as mathematicians. "Poetry may be a perfect medium for presenting ideas of infinity," say Marcia Birken and Anne C. Coon in *Patterns in Mathematics and Poetry,* [4, p 180], pointing to the famous stanza by William Blake as a familiar example: "To see a world in a grain of sand … and eternity in an hour" [5].

In spite of Zeno's example, the mathematicians of ancient Greece were not keen on infinity. They *were* interested in how far you can count and how to handle very large numbers, and they coined the term "sand hundreds" for an inconceivably great quantity [18, p 74]. In his work, *The Sand Reckoner,* Archimedes claimed that there were indeed numbers large enough to count all the sand in the world, and indeed in the whole universe—an audacious claim at the time. However, he considered that this huge number would still be finite [15, loc. 453]. Nevertheless the process he developed for counting such large numbers (his "myriads of myriads") laid the basis for counting systems that have no greatest number. Today, we are quite comfortable with the idea of numbers that go on forever. However, the infinities of the real world are different from the infinities of mathematics. We have more numbers than we will ever need to count anything in the universe; the days and years of our lives, however long, will always be 'small' compared with the endlessness of integers.

A century before Archimedes, the philosopher Eubulides had also used the counting of sand grains as the basis for his *sorites* paradox. It is often stated along the following lines: if you have one grain of sand, it's definitely not a 'heap'. Adding a grain doesn't turn it into a heap. If you keep adding, you will clearly have a heap eventually—but there is no point at which repeating the act of adding (or subtracting) a single grain has changed its status. This is essentially a question of defining 'large' and 'small' [11].

Eubulides' Paradox: Sand Reckonings

How many grains of sand are needed for a heap?
One is not enough, nor three.
Where does *small* become *large*? A beach
has many grains—we will agree

that number's large, however small a stretch
it makes for spades and pails and castles.
Sand hundreds. That's the name the ancient Greeks
coined for the uncountables.
But any number's small when we compare
its span (however vast) with all
infinity beyond it, numberless and fair
with castles we can never reach.
    Sand hundreds trickle quickly through
    my fingers—never days enough for beach
                and you.

— Alice Major [13]

The sonnet form used in these last two poems often seems a natural one with which to explore a paradoxical idea. Its measured qualities and traditional structure of argument/counter-argument/conclusion echo the long history of logical debate these ancient riddles have inspired.

2. The loops of consciousness

Paradox also appeals to poets like me because it resonates with the constantly looping nature of consciousness and language. Take the liar's paradox, another ancient staple of logic and philosophy (which was also attributed to Eubulides). To say, "This sentence is false," leads to the situation where a binary true-or-false assessment of the statement sends us into an unresolvable whirl. Kurt Gödel used a variation of this paradox ("This statement is unprovable") in his First Incompleteness Theorem, to demonstrate that axiomatic systems are inherently limited—that they contain statements that cannot be either proved or disproved within the system [15, loc. 3280].

This is where poetry and math part company to a degree. For mathematicians and physicists, such contradictions tend to be a problem, something that needs to be resolved if a discipline is going to move forward. Mathematicians have to re-think things when they come up against a contradiction like the Koch snowflake, where a perimeter can approach infinity while enclosing a finite area [17]. However human consciousness manages such binary situations all the time. Our capacity to believe (and feel) contradictory things simultaneously seems fundamental to how our brains are structured—the logic gates of our neurons are not binary. They fire on the basis of weighted averages. In the human system, the liar's paradox could easily be, well, true. Or false. Or both.

the liar's paradox

all lovers are liars
I am a lover

old snake-in-the-grass ourobouros

chasing the tail of falsehood
into the mouth of truth

    I love you/love you not
    I could do without you
    very well/am braided to you
    like roots to soil

I say all this with sincerity
wonder where truth stops
on the hoop

tail-in-the-mouth   truthteller serpent
father of lies   lies in the grass
beast with one back   and two minds

    I need you/need you not
    think of life without you/cannot
    imagine it

I am a lover and lie
only when inevitable

          — Alice Major [13]

3. Waves on a roll

Paradoxes have morphed and faded in and out of view over the past two millennia. Some, like the Cartographer's Paradox, get resolved because new information clarifies the logic. (This particular puzzle emerged from the experience of Magellan's crew on their first journey to circumnavigate the globe. In spite of carefully noting the days throughout their journey, they arrived back a day 'early' [15, loc. 1909]—a paradox that would be resolved by the International Date Line.) Other ancient riddles, like the Liar's Paradox, continue to generate new ideas. And new paradoxes turn up all the time as math is applied to the constraints of the 'real' world.

The most stunning for us to absorb over the past century has been wave-particle duality [12, p 49-52], the realization through experiment and theory that everything in the universe seems to have a double, contradictory nature. Sometimes, an electron acts like a continuous wave; at other times, it acts like a discrete particle. Which aspect pops up in any given situation depends on the experimental set-up you use to observe it and the mathematical tools you have used to model it. Wave/particle duality has been deeply confusing and stimulating to physics, and it is essentially (once again) the issue of continuity vs. discontinuity—whether the world is one thing or many things. Which just happens to be the same question that Zeno and his mentor Parmenides were exploring [3, p 20]. (They came down adamantly on

the side of 'one thing' and would likely have approved of discoveries like quantum entanglement [1, pages x-xiii]).

To a poet, once again, this paradox of wave/particle duality reflects something central about living in the world. It seems that everything about our experience can be digitized and pixillated, and yet we feel the whole thing is continuously linked.

Time/text

Time comes quantized
in little books   pocked
with fifteen-minute intervals
that mark my progress through the day —
meetings, tasks, assignments.

Niggling book of kells
spelling out the duties
and services
peculiar to each hour.

They stack up in my desk drawer
beside month-end reports —
proof that I existed in July
of 1990, attended meetings,
ate lunch, wrote a letter. I reorder
from companies called
Daytimer
TimeText

Until I turn a page and find it
    blank
seamless, unrecorded —
a day that has slipped away whole
and entire. For all I know, it was a day
I met with the universe,
illuminated a manuscript
with a single symbol —
    Ω *Omega*, perhaps, or
    ∞ *Eternity*

— Alice Major [14]

To return to a childhood puzzle for my final poem of this paper: Einstein's theory of relativity was deeply paradoxical to me when I first encountered it in Martin Gardner's popularization, *Relativity for the Million* [9]. Here, I mean 'paradox' in the way it is often used in ordinary speech—something that seems counter-intuitive or just plain impossible. Einstein's equations are not paradoxical in one vital sense: his

premises lead to conclusions that are fully and logically consistent. But the twin paradox still boggled my mind. If a pair of twins is separated, and one sent on a space ship at light speed around a distant star, the travelling twin will arrive back home to find the other much older, because the accelerating twin experiences time differently [6, page 59-67].

This thought experiment does not actually lead to loopiness. *Real* time travel would create a genuine paradox: what might happen to the present if we could go back in time and change the past? The meeting of the twins on the landing pad does not result in that kind of contradiction. However, their story does what paradoxes should do: highlight how an argument must follow consistently from its premises. In this case, our usual premise was wrong (that time is a constant background passing at the same speed for everyone) and Einstein's mathematical logic of abandoning it led to surprising conclusions.

Regardless of the mathematical logic, the twins do seem truly paradoxical: born at the same time, they are the same age and yet they are not. The spaceship twin's experience during the flight to and from Alpha Centauri is not of the decades that her earthbound sister is living through. She has had fewer years of heartbeats and breaths, of chemical reactions in her cells and thoughts in her brain. Outwardly they have aged at different speeds, but at a deeper level, they experience the flow of time in exactly the same way.

Twin paradox

I am your twin. You, the starry-eyed young woman
in the heart department, newly engaged, exploring
the curvature of future at light speed,
wearing your fine skin stretched
on its delicate frame.

It is as though you had accelerated in a rocket ship,
travelling to arc around a distant star
and returned to find me,
your twin, with time
puddled at my toes.

While your clock was stretching out its tick-tock metronome
I was making all those astronaut decisions
that swing open now before you
like the hatch of a landing capsule
on an unlived world.

Your young face reflects mine, but our mirror symmetry
is wrinkled by time's stretch marks. You greet me
on this green-again planet
without recognition.
That will come.

— Alice Major [13]

Throughout the recorded history of human thought, paradox has been a sign that we have to look hard for explanations. When we encounter one, we feel there must be some hidden variable that will clarify how continuous behavior arises from discrete bits, or how logic leads to illogical conclusions. We feel the contradictions must lie in something about *us*—our perceptual systems or the language we are using to describe the world. If our equations for the same thing are now wave-like, now particle-like, well underneath there must be some reality that merges the two and we need to go on trying to understand it. This kind of exploration underlies both mathematics and poetry, as we struggle to express concepts as completely and accurately as our languages of symbol or speech will allow.

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